The Common Core State Standards for Mathematics in Grades 9-12

June 29, 2011

The Center on Instruction is operated by RMC Research Corporation in partnership with the Florida Center for Reading Research at Florida State University; Instructional Research Group; Lawrence Hall of Science at the University of California-Berkeley; the Texas Institute for Measurement, Evaluation, and Statistics at the University of Houston; and The Meadows Center for Preventing Educational Risk at the University of Texas at Austin.

The contents of this PowerPoint were developed under cooperative agreement S283B050034 with the U.S. Department of Education. However, these contents do not necessarily represent the policy of the Department of Education, and you should not assume endorsement by the Federal Government.

The Center on Instruction requests that no changes be made to the content or appearance of this product.

To download a copy of this document, visit www.centeroninstruction.org.

2011

THE COMMON CORE STATE STANDARDS FOR MATHEMATICS IN GRADES 9-12

Fred Dillon

dillonsea@att.net

Why do we have the Common Core?

"Notable in the research base for these standards are conclusions from TIMSS and other studies of high-performing countries that the traditional US mathematics curriculum must become substantially more coherent and more focused in order to improve student achievement in mathematics. To deliver on the promise of common standards, the standards must address the problem of a curriculum that is 'a mile wide and an inch deep.' The draft Common Core State Standards for Mathematics are a substantial answer to this challenge."

Common Core State Standards Initiative, *Common Core State Standards for Mathematics*, (http://www.corestandards.org/, 2010) 3.

Status of the Common Core

- As of today, 43 states and the District of Columbia have adopted the Common Core State Standards for Mathematics. Only Alaska, Minnesota, Montana, Nebraska, North Dakota, Texas and Virginia have not signed on to CCSSM. Washington has adopted provisionally
- Two Consortia are developing assessments for the CCSSM (more later)
- Textbooks are already being adapted and written to address the CCSSM

Benefits from the Common Core

- Development of common assessments
- Policy and achievement comparisons across states and districts
- Development of curriculum, professional development and assessments through collaborative groups

Benefits of the Common Core

- Common learning goals for all students
- Coherence
- Focus

CCSSI, 3.

Coherence

- The CCSSM are articulated over time reflecting the sequential (hierarchical) nature of mathematics
- These Standards stress conceptual and procedural understandings of key ideas equally
- Basic organizing principles are used as an organizing structure

CCSSI, 3-4.

Focus

- Addresses the complaint of the "mile wide, inch deep" mathematics curriculum
- Fewer standards that have clarity and specificity – that is the standards are not broad based statements about goals
- Understanding mathematics is the key goal

CCSSI, 3-4.

Reading the CCSSM

The CCSSM are composed of:

- Standards
- Clusters
- Domains

Reading the CCSSM

Standards – What students understand and should be able to do. Related standards will increase in complexity over the years

4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.**

CCSSI, 5, 64.

Reading the CCSSM

Clusters – Groups of related standards.

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.*
   a. Factor a quadratic expression to reveal the zeros of the function it defines.
   b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
   c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression* 1.15^t *can be rewritten as* $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*
4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.**

CCSSI, 5, 64.

Reading the CCSSM

Domains – Larger groups of related standards. These are the big ideas that connect across topics

Seeing Structure in Expressions A-SSE

CCSSI, 5, 64.

Reading the CCSSM

All three of these are incorporated into the conceptual strands, such as Geometry

CCSSI, 5.

Reading the CCSSM

Interpreting Functions F-IF

Analyze functions using different representations

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*

   a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

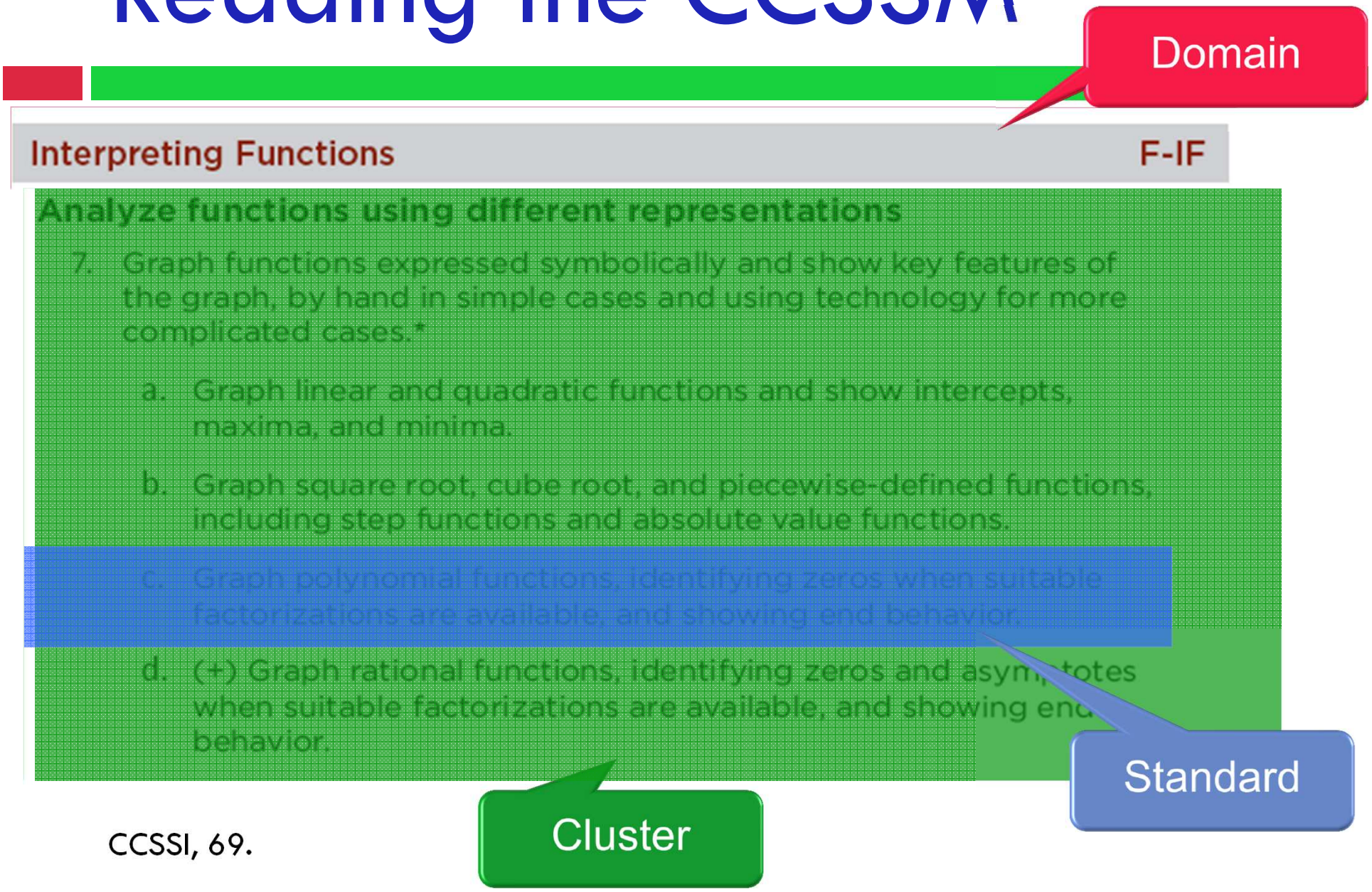
   b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

   c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

   d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

CCSSI, 69.

Reading the CCSSM



CCSSI, 69.

Reading the CCSSM

Interpreting Functions F-IF

Analyze functions using different representations

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*

   a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

   b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

   c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

   d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

Modeling

Regular Standard

STEM

CCSSI, 69.

Standards for Mathematical Practice

The CCSSM Mathematical Principles align closely with the National Council of Teachers of Mathematics goals for students

Standards for Mathematical Practice

The Common Core State Standard Practice Statement

Principles and Standards for School Mathematics

Focus in High School Mathematics

Reasoning and Sense Making

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.

CCSSI, 6-12.

Standards for Mathematical Practice

2. Reason abstractly and quantitatively.

CCSSI, 6-12.

Standards for Mathematical Practice

3. Construct viable arguments and critique the reasoning of others.

CCSSI, 6-12.

Standards for Mathematical Practice

4. Model with mathematics

CCSSI, 6-12.

Standards for Mathematical Practice

5. Use appropriate tools strategically

CCSSI, 6-12.

Standards for Mathematical Practice

6. Attend to precision

CCSSI, 6-12.

Standards for Mathematical Practice

7. Look for and make use of structure

CCSSI, 6-12.

Standards for Mathematical Practice

8. Look for and express regularity in repeated reasoning.

CCSSI, 6-12.

Conceptual Categories

- Number and Quantity
- Algebra
- Functions
- Modeling
- Geometry
- Statistics and Probability

Numbers and Quantity

- Extend the Real Numbers to include work with rational exponents and study of the properties of rational and irrational numbers
- Use quantities and quantitative reasoning to solve problems.

CCSSI, 59-61.

Numbers and Quantity

Required for higher math and/or STEM

- Compute with and use the Complex Numbers, use the Complex Number plane to represent numbers and operations
- Represent and use vectors
- Compute with matrices
- Use vector and matrices in modeling

CCSSI, 59-61.

Algebra and Functions

- Two separate conceptual categories
- Algebra category contains most of the typical "symbol manipulation" standards
- Functions category is more conceptual
- The two categories are interrelated

CCSSI, 62-66.

Algebra

- Creating, reading, and manipulating expressions
  - Understanding the structure of expressions
  - Includes operating with polynomials and simplifying rational expressions
- Solving equations and inequalities
  - Symbolically and graphically

CCSSI, 62-66.

Algebra

Required for higher math and/or STEM

- Expand a binomial using the Binomial Theorem
- Represent a system of linear equations as a matrix equation
- Find the inverse if it exists and use it to solve a system of equations

CCSSI, 62-66.

Functions

- Understanding, interpreting, and building functions
  - Includes multiple representations
- Emphasis is on linear and exponential models
- Extends trigonometric functions to functions defined in the unit circle and modeling periodic phenomena

CCSSI, 67-71

Functions

Required for higher math and/or STEM

- Graph rational functions and identify zeros and asymptotes
- Compose functions
- Prove the addition and subtraction formulas for trigonometric functions and use them to solve problems

CCSSI, 67-71

Functions

- Required for higher math and/or STEM
- Inverse functions
  - Verify functions are inverses by composition
  - Find inverse values from a graph or table
  - Create an invertible function by restricting the domain
  - Use the inverse relationship between exponents and logarithms and in trigonometric functions

CCSSI, 67-71

Modeling

Modeling has no specific domains, clusters or standards. Modeling is included in the other conceptual categories and marked with a asterisk.

CCSSI, 72-73

Modeling

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Technology is valuable in modeling.

A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object.

CCSSI, 72-73

Modeling

- Planning a round-robin basketball tournament for 9 teams
- Analyzing velocity rates of a roller coaster
- Modeling radioactive decay, exponential population growth or value of a car over time

Geometry

Congruence similarity and symmetry are approached by way of geometric transformations

CCSSI, 74-78

Geometry

- Understanding congruence
- Using similarity, right triangles, and trigonometry to solve problems

CCSSI, 74-78

Geometry

- Circles
- Expressing geometric properties with equations
  - Includes proving theorems and describing conic sections algebraically
- Geometric measurement and dimension
- Modeling with geometry

CCSSI, 74-78

Geometry

Required for higher math and/or STEM

- Non-right triangle trigonometry
- Derive equations of hyperbolas and ellipses given foci and directrices
- Give an informal argument using Cavalieri's Principal for the formulas for the volume of solid figures

CCSSI, 74-78

Statistics and Probability

- Analyze single and two variable data
- Understand the role of randomization in experiments
- Make decisions, use inference and justify conclusions from statistical studies
- Use the rules of probability

CCSSI, 79-83

CCSSM 9-12

Questions?

Pathways for High School

- Appendix A – The Pathways for the Traditional Sequence, the Integrated Sequence
- Accelerated Sequences that begin High School mathematics in the Middle School for each

Common Core State Standards Initiative, *Common Core State Standards for Mathematics Appendix A: Designing High School Courses Based on the Common Core State Standards*, , (http://www.corestandards.org/, 2010) 2-3, 15, 27, 36, 51, 61, 72, 80-81, 92.

Traditional Pathway

- An approach typically seen in the United States – two algebra courses, a geometry course and probability and data analysis included
- A fourth year course is not specified

CCSSI, *Appendix A*, 15, 27, 36.

Accelerated Traditional

- A compacted version with no content omitted
- Students may complete the contents of 7th grade, 8th grade and Algebra I by the end of grade 8
- The K-7 CCSSM should prepare students for algebra in 8th grade, but some 8th grade standards are moved to grade 7 to make the Algebra course more manageable

CCSSI, *Appendix A*, 92.

Integrated Pathway

- An approach seen outside the United States – three courses that include number, algebra, geometry, probability and statistics
- A fourth year course is not specified

CCSSI, *Appendix A*, 51, 61, 72.

Accelerated Integrated

- A compacted version with no content omitted
- Students may complete the contents of 7th grade, 8th grade and Algebra I by the end of grade 8
- The K-7 CCSSM should prepare students for algebra in 8th grade, but some 8th grade standards are moved to grade 7 to make the Mathematics course for grade 8 more manageable

CCSSI, *Appendix A*, 92.

All Pathways

- Strategic use of technology is expected in all work
- Support for all students
- Course is dependent on student needs. A variety of courses should be available

What should the fourth year be?

- Precalculus
- Advanced Placement Calculus
- Advanced Placement Statistics
- The CCSSM has clear expectations for this traditional route

Michael Shaughnessy, "AP Calculus: Too Much of a Good Thing?", Advanced Placement Calculus Reading (Kansas City, MO), June 17, 201.

Reports from the MAA/NCTM MUTUAL CONCERNS Committee

- Calculus is not for everyone—and, there are lots of flavors of AP calculus out there. AP students can be ill prepared to continue in calculus in college. (David Bressoud, *The Rocky Transition From High-School Calculus* Jan. 2010, The Chronicle)

- Endless Algebra is the Deadly Pathway from High School Mathematics to College Mathematics (Shaughnessy, NCTM President's Message, Feb., 2011)

Shaughnessy, "AP Calculus"

What are some alternative transitions?

"There is no reason why statistics, linear algebra, geometry, or discrete mathematics cannot be used instead of calculus as a bridge to higher-level mathematics." (Bressoud, 2010)

Shaughnessy, "AP Calculus"

QUANT: Quantifying Uncertainty and Analyzing Numerical Trends

- **Content:** data analysis, combinatorics, probability, and statistical reasoning
- **Pedagogy:** selecting and enacting cognitively demanding instructional tasks
- **Technology:** data collection devices, spreadsheets, and interactive statistical software

Developed by: Dr. Greg Foley, Ohio University and Dr. Tom Butts, UT Dallas

Shaughnessy, "AP Calculus"

Modspar:
Modeling and Spatial Reasoning

- **Content:** discrete, continuous, and geometric modeling as well as spatial reasoning
- **Pedagogy:** creating and using cognitively demanding student assessments
- **Technology:** computer algebra, graphing utilities, and Euclidean, spherical, and 3D dynamic geometry

Developed by: Dr. Greg Foley, Ohio University and Dr. Tom Butts, UT Dallas

Shaughnessy, "AP Calculus"

CRAFTY –Curriculum Renewal Across the First Two Years of College

Reports examined needs of client disciplines: e.g., biology, chemistry, economics, engineering, physics, etc.

- More emphasis on modeling.
- Consideration of multivariate topics.
- Computational skills useful in other fields.
- Units, scaling, dimensional analysis.
- Only the 'fundamental and applicable results from calculus."

NOT:

- non-trivial algebraic manipulations such as integration by parts
- proofs
- use of graphing calculators

Shaughnessy, "AP Calculus"

Formal Statistics Course

- Not necessarily Advanced Placement
- College Statistics enrollment is increasing
- College Calculus enrollment is flat
- AP Statistics is one of the fastest growing subject areas, ever, for the College Board

Shaughnessy, "AP Calculus"

GAISE Reports

Guidelines for Assessment and Instruction of Statistics Education – American Statistical Society

http://www.amstat.org/education/gaise/

GAISE Report

- How to obtain or generate data
- How to graph the data as first step in analysis
- How to interpret numerical summaries and graphical displays
- How to make appropriate use of statistical inference

GAISE Report

- How to communicate the results of a statistical analysis.
- How to interpret statistical results in context.
- How to critique news stories and journal articles that include statistical information
- And, when to call for help from a statistician!

CCSSM 9-12

Questions?

Assessment

"…Some of the highest priority content for college and career readiness comes from Grades 6-8. This body of material includes powerfully useful proficiencies such as applying ratio reasoning in real-world and mathematical problems, computing fluently with positive and negative fractions and decimals, and solving real-world and mathematical problems involving angle measure, area, surface area, and volume." CCSSI, 84.

Assessment

"Because important standards for college and career readiness are distributed across grades and courses, systems for evaluating college and career readiness should reach as far back in the standards as Grades 6-8. It is important to note as well that cut scores or other information generated by assessment systems for college and career readiness should be developed in collaboration with representatives from higher education and workforce development programs, and should be validated by subsequent performance of students in college and the workforce." CCSSI, 84.

PARCC

- The Partnership for Assessment of Readiness for College and Careers
- 25 States working collaboratively to develop a common set of K-12 assessments
- States include California, Florida, Illinois, New York
- Partnered with Achieve

PARCC, "The Partnership for Assessment of Readiness for College and Careers", COMAP Curriculum and Assessment Conference, (Arlington, VA) April 2011,3.

PARCC

- K-12 educators and education leaders involved in development
- More than 200 institutions are assisting and will set the college-ready cut scores to place incoming freshmen
- Formative and summative tools are planned
- Partnership Resource Center with sample tasks, rubrics and released items

PARCC, "The Partnership for Assessment of Readiness for College and Careers", April 2011, 16.

PARCC

Beyond creating assessments, PARCC wants to create:

- Content frameworks to guide educators
- Model instructional units anchored around a PARCC assessment component
- Sample Assessment Tasks that mirror tasks on PARCC assessments
- Professional Development modules
- College ready tools such as model 12th grade bridge courses

PARCC, "The Partnership for Assessment of Readiness for College and Careers", April 2011, 15.

PARCC Survey Link

- https://spreadsheets.google.com/viewform?hl=en&formkey=dDNndEh2TnVaREdNU0FHTEtrNmt3Z2c6MQ#gid=0

Smarter Balanced Assessment Consortium

- 29 States including Michigan, North Carolina, Oregon, Washington
- A system of assessments based on "evidence-based design"
- Assessments are structured to continually improve teaching and learning

Joe Willhoft, Tim Kurtz, "Overview of the SMARTER Balanced Assessment Consortium", COMAP Curriculum and Assessment Conference, (Arlington, VA) April 2011

Smarter Balanced Assessment Consortium

- Adaptive summative assessments benchmarked for college and career readiness
- Flexible interim assessments
- Formative tools
- Includes performance tasks

Joe Willhoft, Tim Kurtz, “Overview”

Moving Forward Together

Report from the COMAP conference held in April, 2011

http://www.mathismore.net/resources/MovingForward/index.html

Moving Forward Together
Recommendations

- Ensure that the Standards for Mathematical Practice are embedded in the assessments.
- Focus attention on content changes at the middle grades.
- Design the PARCC "through-course" assessments to support teaching and learning by facilitating multiple modes of content delivery.
- Assist SBAC in the creation of scoring categories, sub-scores on constructs, and tagging systems to ensure that valid information is reported to teachers, parents, and students.

Michael Shaughnessy, *President's Corner: Assessment and the Common Core State Standards: Let's Stay on Top of It!, http://www.nctm.org/about/content.aspx?id=30169, June 2011.*

Moving Forward Together

- Support long-term sustainability of assessments and an evidence-based approach to revisions to ensure that they are appropriate.
- Enlist the help of curriculum developers in the creation of instructional tools, including content frameworks, model instructional units, formative assessment tools, and resources.
- Request and lobby for policy-level changes to lengthen the timeline and process of implementing the assessments, given the complexity of the task.
- Urge the Council of Chief State School Officers (CCSSO) to create and communicate a governing structure for current and future work on the CCSS.

http://www.nctm.org/about/content.aspx?id=30169

What else is there?

- Making It Happen – NCTM publication
- Joint Committee of NCTM, National Council of Supervisors of Mathematics (NCSM), the Association of State Supervisors of Mathematics (ASSM), and the Association of Mathematics Teacher Educators (AMTE), CCSSO, PARCC and SMARTER

What else is there?

- NCTM website
  - Brief PowerPoints on each gradeband
  - E-seminars on different gradebands and what NCTM can do to help educators with the CCSSM
  - Joint Committee statement about CCSSM and its implementation

CCSSM 9-12

Fred Dillon

dillonsea@att.net

CENTER ON INSTRUCTION

To access materials and the archive of this webinar

www.centeroninstruction.org

For questions or requests for assistance

COI-Info@rmcres.com